Glue-on AdS holography for $T\bar{T}$-deformed CFTs

Wen-Xin Lai 赖文昕

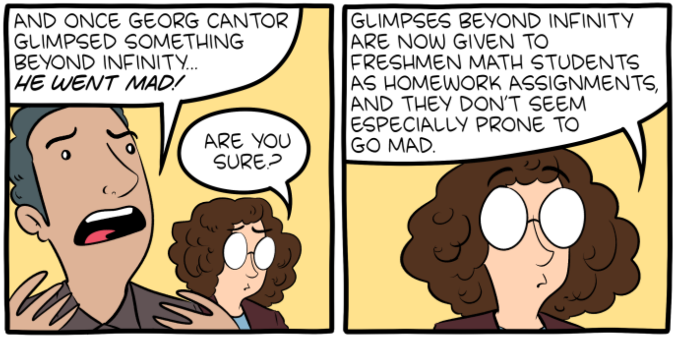
in collaboration with

Luis Apolo, Peng-Xiang Hao 郝鹏翔 and Wei Song 宋伟

arXiv:2303.04836

Yau Mathematical Sciences Center, Tsinghua

2023/09/14 @ BIMSA



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1 Warmup: Pure Einstein gravity in AdS_3 / CFT_2

Maldacena, hep-th/9711200 [1]

$$\text{Strings on } AdS_{d+1} \text{ background} \equiv \text{Conformal Field Theory } CFT_d$$
$$\textit{asympt. } AdS_{d+1} \text{ Gravity} \equiv \text{Large } N \text{ } CFT_d \text{ at } \textit{asympt.} \text{ boundary}$$

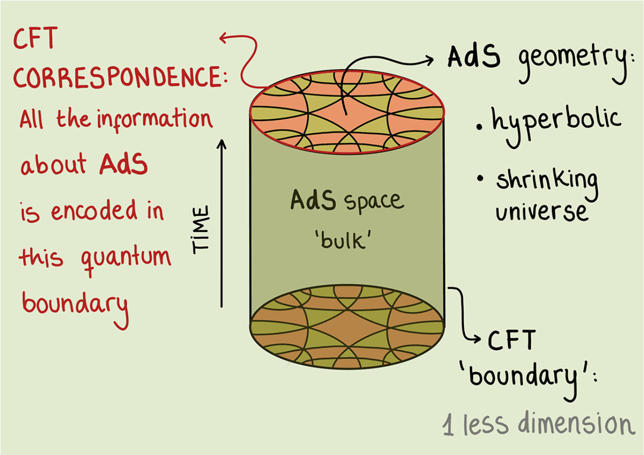


Figure 1: AdS_3/CFT_2 cartoon
Image courtesy: *Aldegunde*, 2022 [2]

AdS/CFT — a model of quantum gravity

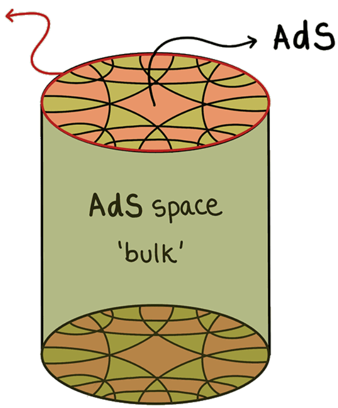
Maldacena, hep-th/9711200 [1]

$$\text{Strings on AdS}_{d+1}\text{ background} \equiv \text{Conformal Field Theory CFT}_d$$
$$\textit{asympt.}\ \text{AdS}_{d+1}\text{ Gravity} \equiv \text{Large } N \text{ CFT}_d \text{ at } \textit{asympt.}\text{ boundary}$$

- A model of QUANTUM GRAVITY
- ... originating from STRING THEORY
- ... seemingly UNIVERSAL / UBIQUITOUS
  - Model agnostic: $\text{AdS}_5 \times S^5$, $\text{AdS}_3 \times S^3 \times T^4$, symmetric orbifolds, minimal models, "monsters", ... *Maldacena, Witten, GKP, ABJM, Gaberdiel, and many many more*
  - Our work: pure Einstein gravity in AdS_3 / CFT_2 ($d = 2$).

AdS_3/CFT_2 — gravity trapped in a cylinder

Aharony, Gubser, Maldacena, Ooguri & Oz, hep-th/9905111 [3]



Aldegunde, 2022 [2]

- AdS_3 metric, $\rho \sim z^2 \sim r^{-2}$

$$ds^2 = \ell^2 \frac{d\rho^2}{4\rho^2} + \left(\frac{1}{\rho} g_{ij}^{(0)} + g_{ij}^{(2)} + \rho\, g_{ij}^{(4)} + \cdots \right) dx^i dx^j \tag{1}$$

boundary at $\rho \to 0$, $x^i \sim \varphi, t$, $i = 1, 2$

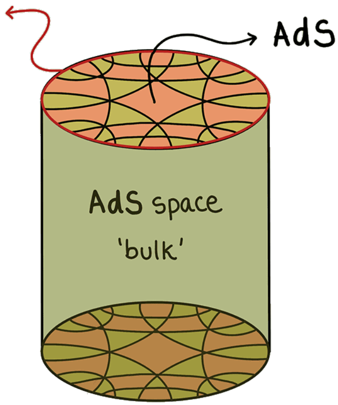
- Surprisingly the expansion simply terminates at $\rho\, g_{ij}^{(4)}$ for pure Einstein gravity! Moreover,

$$g_{ij}^{(4)} = \frac{1}{4} g_{ik}^{(2)} g^{(0)\,kl} g_{lj}^{(2)} \tag{2}$$

Fefferman & Graham, 0710.0919 [4] and *Banados*, hep-th/9901148 [5]

AdS_3/CFT_2 — **simple realization of holography**

Banados, Teitelboim & Zanelli, hep-th/9204099 [6]



Aldegunde, 2022 [2]

- AdS_3 metric for $\rho > 0$,
  Banados, hep-th/9901148 [5]

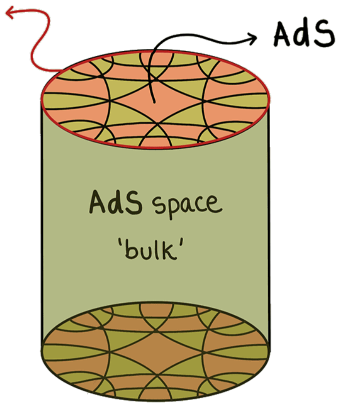
$$ds^2 = \ell^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{\big(du + \rho\, \bar{\mathcal{L}}(v)\, dv\big)\big(dv + \rho\, \mathcal{L}(u)\, du\big)}{\rho} \right)$$

  $g^{(2)} \rightsquigarrow \mathcal{L}(u)$, $\bar{\mathcal{L}}(v)$: arbitrary periodic functions,
  where $u, v = \varphi \mp it$, $du\, dv = d\varphi^2 + dt^2$

- A consequence of Einstein equations
  in $(2+1)$ dimensions.
  Too many constraints, too little freedom!

AdS_3/CFT_2 — **simple realization of holography**

Aharony, Gubser, Maldacena, Ooguri & Oz, hep-th/9905111 [3]



Aldegunde, 2022 [2]

- Different **geometries** in the "bulk"

$$ds^2 = \ell^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{\big(du + \rho\, \bar{\mathcal{L}}(v)\, dv\big)\big(dv + \rho\, \mathcal{L}(u)\, du\big)}{\rho} \right)$$

  specified by $\mathcal{L}(u)$, $\bar{\mathcal{L}}(v)$: periodic functions, maps to different **states** in the boundary CFT_2 with stress energy $\langle T(u)\rangle_\psi$, $\langle \bar{T}(v)\rangle_\psi$

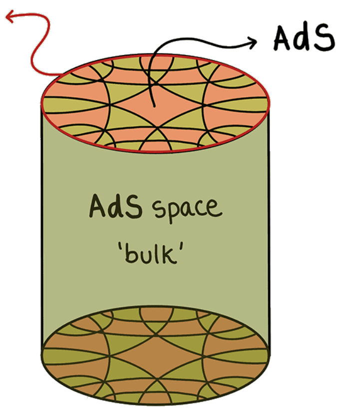
- *Dictionary:* bulk $\leftrightarrow$ boundary quantities:

  – spectrum, symmetry,
  partition functions, correlators, ...

2 Review: $T\bar{T}$ deformation and cutoff AdS

$T\bar{T}$ deformations — motivations

Comments taken from *Cui, Shu, Song & Wang*, 2304.04684 [7]



Aldegunde, 2022 [2]

- Ideally: AdS/CFT
- Reality: non-AdS space and non-CFT
  - possible to extend holographic principle to more general context?
  - attempt: ***deformations*** on both sides in a controllable way

$T\bar{T}$ deformations — definition

Zamolodchikov, hep-th/0401146 [8], revived by *Smirnov & Zamolodchikov*, 1608.05499 [9] and *Cavaglià, Negro, Szécsényi & Tateo*, 1608.05534 [10] *et al*

- Define "$T\bar{T}$" as the flow of action:

$$\partial_\mu I = -8\pi \int d^2x \, T\bar{T} = -\pi \int d^2x \left(T^{ij} T_{ij} - (T^i_i)^2 \right) \tag{3}$$

  For CFT_2, $T\bar{T} = T_{xx} T_{\bar{x}\bar{x}}$, where $x, \bar{x} = \varphi' \mp it'$.

- The stress tensor $T_{ij}(\mu)$ on the right hand side flows with the deformation parametrized by μ.

- This is thus a *differential equation* for a flow, starting from some generic QFT specified by $I(\mu = 0)$. In this work we shall start from CFT_2, but note that $T\bar{T}$ is *irrelevant*: conformal symmetry will be broken.

“Solvable” deformations on the field theory side

Zamolodchikov, hep-th/0401146 [8], revived by *Smirnov & Zamolodchikov*, 1608.05499 [9] and *Cavaglià, Negro, Szécsényi & Tateo*, 1608.05534 [10] *et al*

- **“Solvable”:** the deformed spectrum of $\hat{H}(\mu)$ and $\hat{J}(\mu)$ on a cylinder of radius R can be solved, and it's a simple function
  of the undeformed $E(0)$, $J(0)$:

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu}{R}E(0) + \frac{4\mu^2}{R^4}J(0)^2}\right), \qquad J(\mu) = J(0) \tag{4}$$

  under simple (and reasonable) assumptions (e.g. translation invariance et al).

- **Variants:** suppose the theory has some “components” labeled by w,
  - double-trace $T\bar{T} = (\sum_w T_w)(\sum_w \bar{T}_w)$ (this work)
  - single-trace $T\bar{T}' = \sum_w (T_w \bar{T}_w)$ (e.g. *Cui, Shu, Song & Wang*, 23)

$T\bar{T}$ deformations — properties

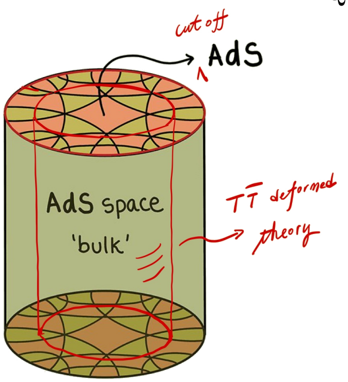
Zamolodchikov, hep-th/0401146 [8]
"Expectation value of composite field T anti-T in two-dimensional quantum field theory"

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu}{R}E(0) + \frac{4\mu^2}{R^4}J(0)^2}\right), \qquad J(\mu) = J(0) \tag{5}$$

- It seems that the $T\bar{T}$ deformation is well-defined non-perturbatively for generic QFT_2, yet it's still mysterious and difficult
  - e.g. correlators: *Cui, Shu, Song & Wang*, 2304.04684 [7], *Kraus, Liu & Marolf*, 1801.02714 [11], and *Cardy*, 1907.03394 [12]
  - "asymptotic fragility": *Dubovsky, Gorbenko & Mirbabayi*, 1706.06604 [13]
- What if we start from a CFT_2, that enjoys a holographic dual to AdS_3...

Cutoff AdS_3 / $T\bar{T}$ **deformed** CFT_2

McGough, Mezei & Verlinde, 1611.03470 [14] – "Moving the CFT into the bulk with $T\overline{T}$"

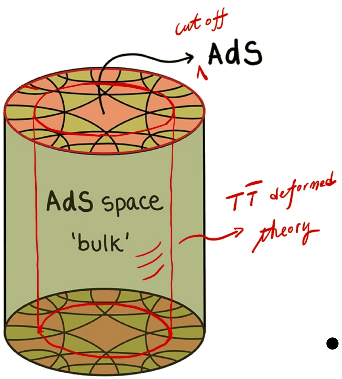


Aldegunde, 2022 [2]

- Question: what is the AdS dual of $T\bar{T}$ deformation?
  - look up *double-trace deformations* in *the dictionary*: *Heemskerk & Polchinski*, 1010.1264 [15]
  - look at properties of $T\bar{T}$: signs of gravity and coarse-graining *Dubovsky, Flauger & Gorbenko*, 1205.6805 [16] and *Dubovsky, Gorbenko & Mirbabayi*, 1305.6939 [17]
- Answer: AdS gravity within a finite Dirichlet wall

Cutoff AdS_3 **— the dictionary**

McGough, Mezei & Verlinde, 1611.03470 [14] – "Moving the CFT into the bulk with $T\overline{T}$"



Aldegunde, 2022 [2]

- Location (radius) of the cutoff surface:

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \tag{6}$$

gets mapped to the deformation parameter μ:

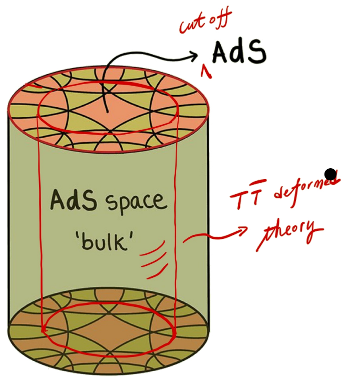
$$\zeta_c = -\frac{c\mu}{3\ell^2} \tag{7}$$

Roughly $\mu \sim 1/r^2$

- Passed many non-trivial tests
- Related to "mixed boundary conditions" by *Guica & Monten*, 1906.11251 [18]

Cutoff AdS_3 — gravity in a box

Title inspired by *Kraus, Monten & Myers*, 2103.13398 [19] – “3D Gravity in a Box”



Aldegunde, 2022 [2]

- **Significance:** gravity inside a finite box with hard Dirichlet walls
  - Dual to some “solvable” no-longer-conformal ft
  - A step towards quantum gravity in reality!
- **Caveat:** the duality only admits $\zeta_c > 0$ so $\mu < 0$

$$\zeta_c = -\frac{c\mu}{3\ell^2} \tag{8}$$

But $T\bar{T}$ itself admits $\mu > 0$ with nice properties.
What is the other side of the duality?

  - For comparison, the proposal of *Guica & Monten*, 1906.11251 [18] admits both signs of μ

3 Proposal: Glue-on AdS holography

Glue-on AdS_3 — analytic continuation

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

- AdS_3 metric has only simple poles with the ρ coordinate:

$$ds^2 = \ell^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{\big(du + \rho\, \bar{\mathcal{L}}(v)\, dv\big)\big(dv + \rho\, \mathcal{L}(u)\, du\big)}{\rho} \right) \tag{9}$$

  thus it admits well-defined analytic continuation
  from $\rho > 0$ to $\rho \in \mathbb{R}$.

- Continuation of the dictionary:

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \in \mathbb{R}, \qquad \zeta_c = -\frac{c\mu}{3\ell^2} \in \mathbb{R} \tag{10}$$

Glue-on AdS_3 / $T\bar{T}$— **updating the dictionary**

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

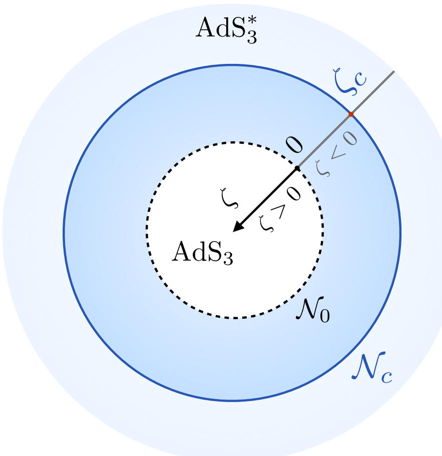


Figure 2: Glue-on AdS_3

Top-down view of the Poincare disk

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \in \mathbb{R}, \qquad \zeta_c = -\frac{c\mu}{3\ell^2} \in \mathbb{R} \tag{11}$$

- What have we done other than copy-pasting?
  - Although the continuation is straightforward, physical quantities diverge as $\rho \to 0$
  - A prescription is required to "renormalize" the divergences

Glue-on AdS_3 / $T\bar{T}$— updating the dictionary

Kraus, Liu & Marolf, 1801.02714 [11] – "Cutoff AdS_3 versus the $T\overline{T}$ deformation"

- Firstly, matching energy momentum (Brown-York) & the flow equations:

$$T_{ij} = \frac{\sigma_{\zeta_c}}{8\pi G}\Big(K_{ij} - K\gamma_{ij} + \frac{1}{\ell|\zeta_c|}\gamma_{ij}\Big), \qquad \sigma_{\zeta_c} = \frac{|\zeta_c|}{\zeta_c} = \pm 1, \tag{12}$$

  The field theory metric $\gamma_{ij} = \zeta_c h_{ij}$, where h_{ij} is the induced metric.

  – γ_{ij} is always positive-definite, while h_{ij} becomes negative-definite for the glue-on region. This discrepancy is the origin of the $|\zeta_c|$.

- $T\bar{T}$ flow is recast geometrically as the i, j components of the Einstein equations. Note that the geometry can be decomposed such that:

$$ds^2 = \frac{1}{\zeta}\gamma_{ij}dx^i dx^j + n_\mu n_\nu dx^\mu dx^\nu \tag{13}$$

Glue-on AdS_3 — beyond the infinity

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

- The geometry is foliated by constant ζ surfaces $\mathcal{N}_\zeta$:

$$\begin{aligned} ds^2 &= \ell^2\left(\frac{d\rho^2}{4\rho^2} + \frac{\big(du + \rho\,\bar{\mathcal{L}}(v)\,dv\big)\big(dv + \rho\,\mathcal{L}(u)\,du\big)}{\rho}\right) \\ &= \frac{1}{\zeta}\gamma_{ij}dx^i dx^j + n_\mu n_\nu dx^\mu dx^\nu, \qquad \frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \end{aligned} \tag{14}$$

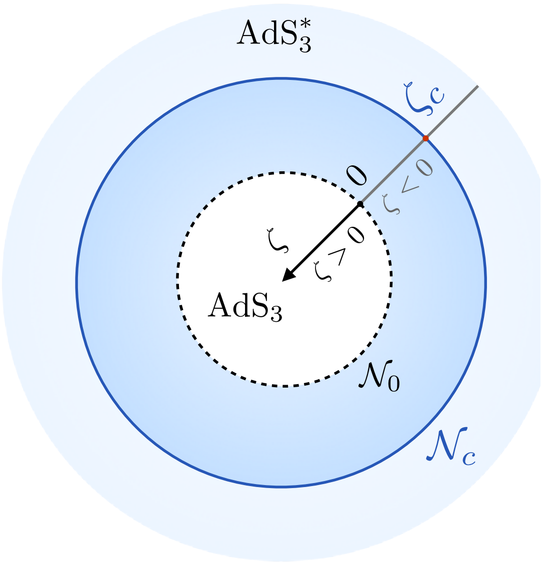
  The $T\bar{T}$ deformed theory lives on $\mathcal{N}_{\zeta_c}$ with metric γ_{ij}.

- Treatment for the singularity at $\zeta \to 0^\pm$: introduce $\mathcal{N}_{\zeta=\pm\epsilon}$ and glue them together ("glue-on"); exclude the $-\epsilon < \zeta < \epsilon$ region until finally sending $\epsilon \to 0$.

- We formally denote the boundary surface by the limit $\mathcal{N}_0 = \mathcal{N}_{0^+} = \mathcal{N}_{0^-}$, though we need to keep track of the asymptotic cutoff ϵ in actual computations.

Glue-on AdS_3 / $T\bar{T}$— **the proposal**

Apolo, Hao, Lai & Song, 2303.04836 [20] – “Glue-on AdS holography for $T\bar{T}$-deformed CFTs”



Cutoff / *glue-on* AdS_{d+1} Gravity $\equiv$ $T\bar{T}$ deformed CFT_d at $\mathcal{N}_{\zeta_c}$

4 Check: $T\bar{T}$ deformed charges & partition functions

$T\bar{T}$ quantities from bulk geometry

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

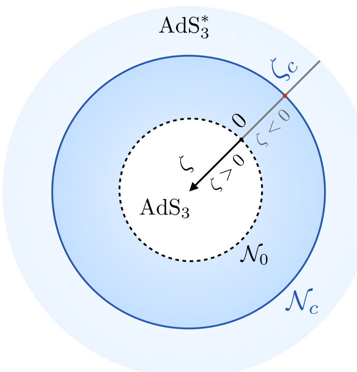


Figure 3: Glue-on AdS$_3$
Top-down view of the Poincaré disk

- AdS/CFT: weak / strong duality
  - *quantum* quantities on the CFT side can be computed by (semi-)classical geometry
  - inherited by the $T\bar{T}$ deformation
- Demanding the extended geometry to be non-singular reproduces bounds on the $T\bar{T}$ deformed theory, e.g.

$$\zeta_c \geq -1 \quad \Leftrightarrow \quad \mu \leq \frac{3\ell^2}{c} \tag{15}$$

$\zeta_c = -1$ is a horizon of the glue-on geometry, where $\det g_{\mu\nu} = 0$.

$T\bar{T}$ spectrum from bulk covariant charges

Kraus, Monten & Myers, 2103.13398 [19] and *Apolo, Hao, Lai & Song*, 2303.04836 [20]

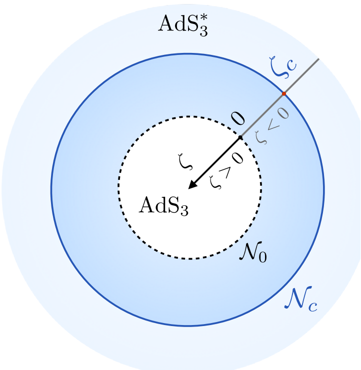


Figure 4: Glue-on AdS_3
Top-down view of the Poincaré disk

- Spectrum: conserved charges of geometries, e.g. black holes *BTZ* [6]
  - computed with the covariant formalism *Iyer & Wald*, gr-qc/9403028 [21] and *Barnich & Brandt*, hep-th/0111246 [22]
  - manifest covariance (diff-invariance) depends on reparametrization redundancies (gauge), so delay gauge fixing until the very end
- We are careful to match the bulk & boundary symmetries at $\mathcal{N}_{\zeta_c}$
- This reproduces $E(\mu)$, $J(\mu)$ with $\mu \in \mathbb{R}$ in (5)

$T\bar{T}$ spectrum from bulk covariant charges

Kraus, Monten & Myers, 2103.13398 [19] and *Apolo, Hao, Lai & Song*, 2303.04836 [20]

- Covariant charges of geometries

$$\delta E(\mu) \equiv \ell^{-1} \delta \mathcal{Q}_{\partial_{t'}},$$
$$\delta J(\mu) \equiv \delta \mathcal{Q}_{\partial_{\varphi'}},$$

- One needs to choose the appropriate boundary coordinates (t', φ'), such that

$$ds_c^2 = \ell^2(-dt'^2 + d\varphi'^2), \qquad \varphi' \sim \varphi' + 2\pi. \tag{16}$$

- This is realized by the *state-dependent* map of coordinates

$$dt' = \sqrt{\big(1 - \zeta_c (T_u + T_v)^2\big)\big(1 - \zeta_c (T_u - T_v)^2\big)}\, dt,$$
$$d\varphi' = d\varphi + \zeta_c \big(T_u^2 - T_v^2\big)\, dt.$$

$T\bar{T}$ thermodynamics from bulk geometry

Giveon, Itzhaki & Kutasov, 1701.05576 [23] and *Apolo, Detournay & Song*, 1911.12359 [24]

- *State-dependent* map of coordinates

$$dt' = \sqrt{\big(1 - \zeta_c (T_u + T_v)^2\big)\big(1 - \zeta_c (T_u - T_v)^2\big)}\, dt,$$
$$d\varphi' = d\varphi + \zeta_c \big(T_u^2 - T_v^2\big)\, dt.$$

- Many consequences: correct charges, signal propagation speed $v'_{\pm} \equiv \pm \frac{d\varphi'}{dt'}$, and thermodynamics when Wick rotated:

$$T_L(\mu)\, T_R(\mu) \leq -\frac{1}{4\pi^2 \ell^2 \zeta_c} = \frac{3}{4\pi^2 c \mu} = T_H(\mu)^2, \quad \mu > 0 \tag{17}$$

  $T_{L,R}$ are temperatures associated with $u', v' = \varphi' \pm t'$.

- T_H is the *Hagedorn temperature*: exceeding T_H corresponds to a complex entropy

5 Check: $T\bar{T}$ partition functions from glue-on AdS

$T\bar{T}$ partition functions from bulk on-shell actions

Caputa, Datta, Jiang & Kraus, 2011.04664 [25]

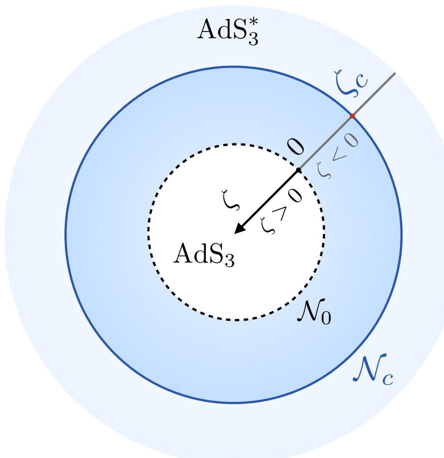


Figure 5: Glue-on AdS_3
Top-down view of the Poincaré disk

- Bulk: weakly coupled gravity, the partition function is approximated by the on-shell gravity action of the dominant saddle:

$$Z_{T\bar{T}}(\mu) = \mathcal{Z}(\zeta_c) \approx e^{-I(\zeta_c)} \tag{18}$$

- $I(\zeta_c)$ is roughly the volume of the geometry, which diverges at $\zeta \to 0^{\pm}$. We apply a natural extension of *holographic renormalization* to remove the divergence.

- $I = -\log \mathcal{Z}$ satisfies the $T\bar{T}$ flow (3).
  This is non-trivial because we've packaged the quantum corrections of the boundary theory.

Comparison with field theory analysis

Datta & Jiang, 1806.07426 [26] and *Apolo, Song & Yu*, 2301.04153 [27]

- **Torus:** Large c, *modular invariance* & sparseness of the "light" spectrum [26, 27], c.f. *Hartman, Keller & Stoica*, 1405.5137 [28]

$$\log Z_{T\bar{T}}(\mu) \approx \begin{cases} -\frac{1}{2}\left(\beta_L + \beta_R\right) R E_{\text{vac}}(\mu), & \beta_L\beta_R > 1, \\ -2\pi^2\left(\dfrac{1}{\beta_L} + \dfrac{1}{\beta_R}\right) R E_{\text{vac}}\left(\dfrac{4\pi^2}{\beta_L\beta_R}\mu\right), & \beta_L\beta_R < 1, \end{cases}$$

- **Sphere:** maximally symmetric, *Donnelly & Shyam*, 1806.07444 [29]

$$\langle T_{ij} \rangle = -\frac{1}{4\pi\mu}\left(1 - \sqrt{1 - \frac{c\mu}{3L^2}}\right)\gamma_{ij}. \tag{19}$$

Sphere partition functions

Donnelly & Shyam, 1806.07444 [29] and *Li*, 2012.14414 [30]

- Given the explicit stress tensor $\langle T_{ij}\rangle \propto \gamma_{ij}$, the flow equations:

$$\partial_\mu \log Z_{T\bar{T}}(\mu) = 8\pi \int d^2x \sqrt{\gamma}\, \langle T\bar{T}\rangle$$
$$-L\,\partial_L \log Z_{T\bar{T}}(\mu) = \int d^2x \sqrt{\gamma}\, \langle T^i_i\rangle$$

- ... admit the general solution with a μ-*independent* integration a:

$$\log Z_{T\bar{T}}(\mu, a) = \frac{c}{3} \log\left[\frac{L}{a}\left(1 + \sqrt{1 - \frac{c\mu}{3L^2}}\right)\right] - \frac{L^2}{\mu}\sqrt{1 - \frac{c\mu}{3L^2}} + \frac{L^2}{\mu}$$

- *Donnelly & Shyam*, 1806.07444 [29] is recovered with $a = \sqrt{c|\mu|/3}$ but one can also take $a = \epsilon$, thus decoupling the energy (RG) scale ϵ with the deformation μ.

Gravitational actions: with counterterms

Donnelly & Shyam, 1806.07444 [29] and *Li*, 2012.14414 [30]

- Gravitational actions, with counterterms:

$$\begin{aligned} I_{\mathcal{M}}(\zeta_1,\zeta_2) &\coloneqq -\frac{1}{16\pi G}\int_{\zeta_1}^{\zeta_2} d\zeta \int d^2x\sqrt{g}\,(R+2\ell^{-2}), \\ I_{\mathcal{N}_\zeta} &\coloneqq -\frac{1}{8\pi G}\int_{\mathcal{N}_\zeta} d^2x\sqrt{h}\,h^{ij}K_{ij} \\ &\quad + \frac{\sigma_\zeta}{8\pi G}\int_{\mathcal{N}_\zeta} d^2x\sqrt{h}\left(\ell^{-1} - \frac{\ell\mathcal{R}[h]}{4}\log|\zeta|\right), \end{aligned}$$

  ... consistent with the Brown-York stress tensor T_{ij},
  fully renormalized at $\zeta \to 0^{\pm}$.

- The lack of diff invariance of the $\log|\zeta|$ counterterm is known to be a reflection of the Weyl anomaly: *Henningson & Skenderis, de Haro, Solodukhin & Skenderis, Papadimitriou.*

Gravitational actions: with counterterms

Caputa, Datta, Jiang & Kraus, 2011.04664 [25] and *Li*, 2012.14414 [30]

- The log counterterm makes a crucial contribution to the on-shell action:

$$
\begin{aligned}
I_{\mathcal{M}}(\zeta_1,\zeta_2) &\coloneqq -\frac{1}{16\pi G}\int_{\zeta_1}^{\zeta_2} d\zeta \int d^2x\sqrt{g}\,(R+2\ell^{-2}),\\
I_{\mathcal{N}_\zeta} &\coloneqq -\frac{1}{8\pi G}\int_{\mathcal{N}_\zeta} d^2x\sqrt{h}\,h^{ij}K_{ij}\\
&\qquad + \frac{\sigma_\zeta}{8\pi G}\int_{\mathcal{N}_\zeta} d^2x\sqrt{h}\left(\ell^{-1}-\frac{\ell\mathcal{R}[h]}{4}\log|\zeta|\right),
\end{aligned}
$$

  It guarantees that the partition function is compatible with the $T\bar{T}$ differential equation — not the case for *Donnelly & Shyam*.

- In general the space of deformed theories are parametrized by (μ, a), where a is the length scale (inverse energy scale). In many contexts they are naturally related: $a = \sqrt{c|\mu|/3}$, but a can be tuned by RG.

6 Summary & future directions

Glue-on AdS_3 / $T\bar{T}$ deformed CFT_2

Apolo, Hao, Lai & Song, 2303.04836 [20] – “Glue-on AdS holography for $T\bar{T}$-deformed CFTs”

$$\text{Cutoff / } \textit{glue-on } \mathrm{AdS}_{d+1} \text{ Gravity } \equiv T\bar{T} \text{ deformed } \mathrm{CFT}_d \text{ at } \mathcal{N}_{\zeta_c}$$

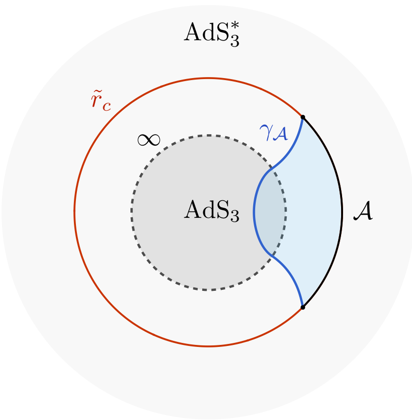


Figure 6: RT for Glue-on AdS_3
Top-down view of the Poincaré disk

- Holographic proposal for $T\bar{T}$-deformed CFTs with $\mu \in \mathbb{R}$.
- As evidence, we show that the $T\bar{T}$ trace flow equation, the spectrum on the cylinder, and the partition function on the torus and the sphere, among other results, can all be reproduced from bulk calculations in glue-on AdS_3.
- We hope to understand the entanglement structure of $T\bar{T}$ deformed theories from bulk geometry.

Glue-on AdS_3 / $T\bar{T}$ **deformed** CFT_2

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

$$\text{Cutoff / } \textit{glue-on } \mathrm{AdS}_{d+1} \text{ Gravity} \equiv T\bar{T} \text{ deformed } \mathrm{CFT}_d \text{ at } \mathcal{N}_{\zeta_c}$$

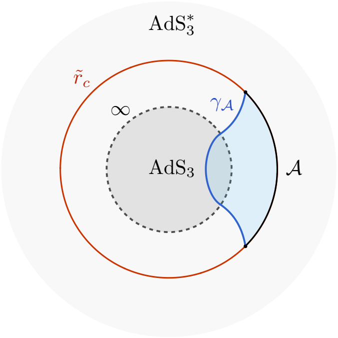


Figure 7: RT for Glue-on AdS_3
Top-down view of the Poincaré disk

- We hope to understand the entanglement structure of $T\bar{T}$ deformed theories from bulk geometry.
  - *Donnelly & Shyam*, 1806.07444 [29] and *Lewkowycz, Liu, Silverstein & Torroba*, 1909.13808 [34]
  - *He & Sun*, 2301.04435 [35], *He, Yang, Zhang & Zhao*, 2305.10984 [36], *Tian*, 2306.01258 [37], and *Hou, He & Jiang*, 2306.07784 [38]

Thank you!

froms Wen-Xin Lai 赖文昕

in collaboration with

Luis Apolo, Peng-Xiang Hao 郝鹏翔 and Wei Song 宋伟

arXiv:2303.04836

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