Glue-on AdS holography for $T\bar{T}$-deformed CFTs

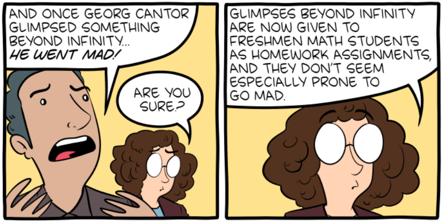
An extended AdS / $T\bar{T}$ duality *(beyond infinity)*

Wen-Xin Lai 赖文昕 *in collaboration with*
Luis Apolo, Peng-Xiang Hao 郝鹏翔 and Wei Song 宋伟

arXiv:2303.04836

Yau Mathematical Sciences Center, Tsinghua

2023/09/14 @ BIMSA



https://smbc-comics.com/comic/cantor

Outline

1. **Warmup:** Pure Einstein gravity in AdS_3 / CFT_2
2. **Review:** $T\bar{T}$ deformation and cutoff AdS
3. **Proposal:** Glue-on AdS holography
4. **Check:** $T\bar{T}$ deformed charges & partition functions
5. **Check:** $T\bar{T}$ partition functions from glue-on AdS
6. Summary & future directions

Warmup: Pure Einstein gravity in AdS_3 / CFT_2

AdS/CFT — holography duality

Maldacena, hep-th/9711200 [2] — "The Large N limit of superconformal field theories and supergravity"

$$\text{Strings on } \mathrm{AdS}_{d+1} \text{ background} \equiv \text{Conformal Field Theory } \mathrm{CFT}_d$$

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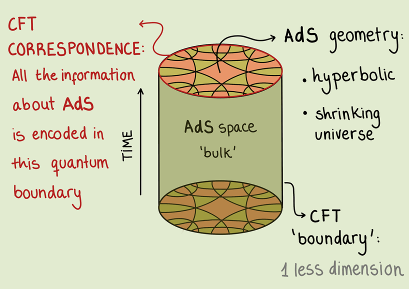

Figure: $\mathrm{AdS}_3/\mathrm{CFT}_2$ cartoon

Image courtesy: *Aldegunde*, 2022 [1]

AdS/CFT — a model of quantum gravity

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- A model of QUANTUM GRAVITY
- ... originating from STRING THEORY
- ... seemingly UNIVERSAL / UBIQUITOUS
  - Model agnostic: $\mathrm{AdS}_5 \times S^5$, $\mathrm{AdS}_3 \times S^3 \times T^4$, symmetric orbifolds, minimal models, "monsters", ... *Maldacena, Witten, GKP, ABJM, Gaberdiel, and many many more*
  - Our work: pure Einstein gravity in AdS_3 / CFT_2 $(d = 2)$.

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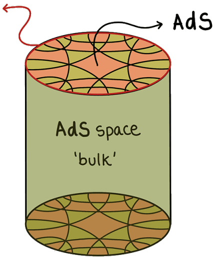
$\mathrm{AdS}_3/\mathrm{CFT}_2$ — gravity trapped in a cylinder

Aharony, Gubser, Maldacena, Ooguri & Oz, hep-th/9905111 [5]



Aldegunde, 2022 [1]

- AdS_3 metric, $\rho \sim z^2 \sim r^{-2}$

$$ds^2 = \ell^2 \frac{d\rho^2}{4\rho^2} + \left(\frac{1}{\rho} g_{ij}^{(0)} + g_{ij}^{(2)} + \rho g_{ij}^{(4)} + \cdots\right) dx^i dx^j \tag{1}$$

boundary at $\rho \to 0$, $x^i \sim \varphi, t$, $i = 1, 2$

- Surprisingly the expansion simply terminates at $\rho g_{ij}^{(4)}$ for pure Einstein gravity! Moreover,

$$g_{ij}^{(4)} = \frac{1}{4} g_{ik}^{(2)} g^{(0)\,kl} g_{lj}^{(2)} \tag{2}$$

Fefferman & Graham, 0710.0919 [3] and *Banados*, hep-th/9901148 [4]

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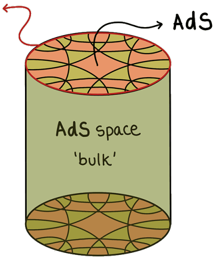

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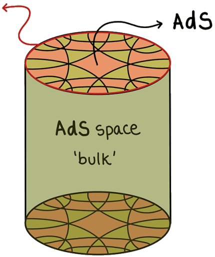
$\mathrm{AdS}_3/\mathrm{CFT}_2$ — simple realization of holography

Banados, Teitelboim & Zanelli, hep-th/9204099 [6]



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$$ds^2 = \ell^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{\left(du + \rho\,\bar{\mathcal{L}}(v)\,dv\right)\left(dv + \rho\,\mathcal{L}(u)\,du\right)}{\rho} \right)$$

$g^{(2)} \rightsquigarrow \mathcal{L}(u)$, $\bar{\mathcal{L}}(v)$: arbitrary periodic functions, where $u, v = \varphi \mp it$, $du\,dv = d\varphi^2 + dt^2$

- A consequence of Einstein equations in $(2+1)$ dimensions.
  Too many constraints, too little freedom!

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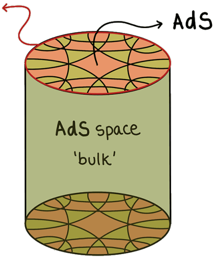

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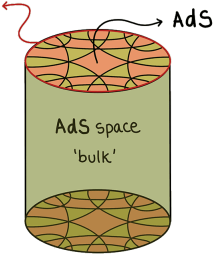

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- Different **geometries** in the "bulk"

$$ds^2 = \ell^2\left(\frac{d\rho^2}{4\rho^2}+\frac{\big(du+\rho\,\bar{\mathcal{L}}(v)\,dv\big)\big(dv+\rho\,\mathcal{L}(u)\,du\big)}{\rho}\right)$$

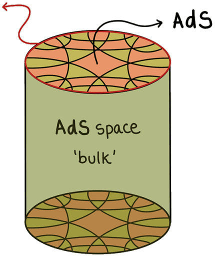
  specified by $\mathcal{L}(u)$, $\bar{\mathcal{L}}(v)$: periodic functions, maps to different **states** in the boundary CFT_2 with stress energy $\langle T(u)\rangle_\psi$, $\langle \bar{T}(v)\rangle_\psi$

- *Dictionary:* bulk $\leftrightarrow$ boundary quantities:
  - spectrum, symmetry, partition functions, correlators, ...

Review: $T\bar{T}$ deformation and cutoff AdS

$T\bar{T}$ deformations — motivations

Comments taken from *Cui, Shu, Song & Wang*, 2304.04684 [7]



Aldegunde, 2022 [1]

- Ideally: AdS/CFT
- Reality: non-AdS space and non-CFT
  - possible to extend holographic principle to more general context?
  - attempt: **deformations** on both sides in a controllable way

$T\bar{T}$ deformations — definition

Zamolodchikov, hep-th/0401146 [8], revived by *Smirnov & Zamolodchikov*, 1608.05499 [9] and *Cavaglià, Negro, Szécsényi & Tateo*, 1608.05534 [10] *et al*

- Define "$T\bar{T}$ " as the flow of action:

$$\partial_\mu I = -8\pi \int d^2x \, T\bar{T} = -\pi \int d^2x \left(T^{ij}T_{ij} - (T^i_i)^2 \right) \tag{3}$$

For CFT_2, $T\bar{T} = T_{xx}T_{\bar{x}\bar{x}}$, where $x, \bar{x} = \varphi' \mp it'$.

- The stress tensor $T_{ij}(\mu)$ on the right hand side flows with the deformation parametrized by μ.
- This is thus a *differential equation* for a flow, starting from some generic QFT specified by $I(\mu = 0)$. In this work we shall start from CFT_2, but note that $T\bar{T}$ is *irrelevant*: conformal symmetry will be broken.

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"Solvable" deformations on the field theory side

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- **"Solvable":** the deformed spectrum of $\hat{H}(\mu)$ and $\hat{J}(\mu)$ on a cylinder of radius R can be solved, and it's a simple function of the undeformed $E(0)$, $J(0)$:

$$E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu}{R}E(0) + \frac{4\mu^2}{R^4}J(0)^2}\right), \qquad J(\mu) = J(0) \tag{4}$$

  under simple (and reasonable) assumptions (e.g. translation invariance et al).

- **Variants:** suppose the theory has some "components" labeled by w,
  - double-trace $T\bar{T} = (\sum_w T_w)(\sum_w \bar{T}_w)$ (this work)
  - single-trace $T\bar{T}' = \sum_w (T_w \bar{T}_w)$ (e.g. *Cui, Shu, Song & Wang*, 23)

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  - "asymptotic fragility": *Dubovsky, Gorbenko & Mirbabayi*, 1706.06604 [13]
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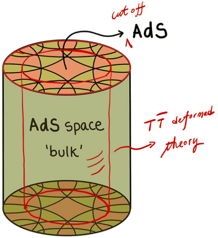
Cutoff AdS_3 / $T\bar{T}$ deformed CFT_2

McGough, Mezei & Verlinde, 1611.03470 [17] – "Moving the CFT into the bulk with $T\bar{T}$"



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- Question: what is the AdS dual of $T\bar{T}$ deformation?
  - look up *double-trace deformations* in *the dictionary*: *Heemskerk & Polchinski*, 1010.1264 [14]
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- Answer: AdS gravity within a finite Dirichlet wall

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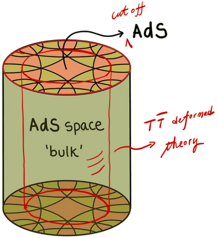

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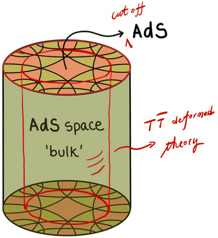

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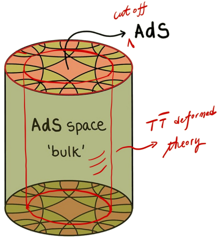

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- Location (radius) of the cutoff surface:

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \tag{6}$$

gets mapped to the deformation parameter μ:

$$\zeta_c = -\frac{c\mu}{3\ell^2} \tag{7}$$

Roughly $\mu \sim 1/r^2$

- Passed many non-trivial tests
- Related to "mixed boundary conditions" by *Guica & Monten*, 1906.11251 [18]

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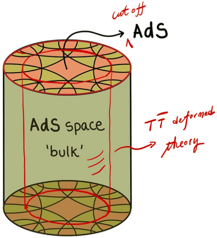

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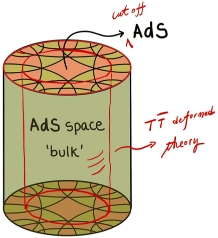

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- **Significance:** gravity inside a finite box with hard Dirichlet walls
  - Dual to some "solvable" no-longer-conformal ft
  - A step towards quantum gravity in reality!
- **Caveat:** the duality only admits $\zeta_c > 0$ so $\mu < 0$

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But $T\bar{T}$ itself admits $\mu > 0$ with nice properties. *What is the other side of the duality?*

  - For comparison, the proposal of *Guica & Monten*, 1906.11251 [18] admits both signs of μ

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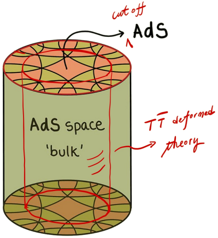

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Proposal: Glue-on AdS holography

Glue-on AdS_3 — analytic continuation

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

- AdS_3 metric has only simple poles with the ρ coordinate:

$$ds^2 = \ell^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{\big(du + \rho\, \bar{\mathcal{L}}(v)\, dv\big)\big(dv + \rho\, \mathcal{L}(u)\, du\big)}{\rho} \right) \tag{9}$$

  thus it admits well-defined analytic continuation from $\rho > 0$ to $\rho \in \mathbb{R}$.

- Continuation of the dictionary:

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \in \mathbb{R}, \qquad \zeta_c = -\frac{c\mu}{3\ell^2} \in \mathbb{R} \tag{10}$$

Glue-on AdS_3 / $T\bar{T}$— updating the dictionary

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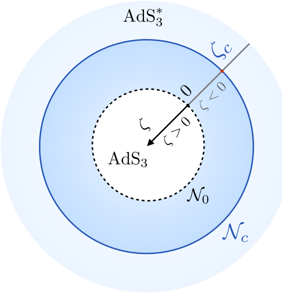

Figure: Glue-on AdS_3
Top-down view of the Poincaré disk

$$\frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \in \mathbb{R}, \qquad \zeta_c = -\frac{c\mu}{3\ell^2} \in \mathbb{R} \qquad (11)$$

- What have we done other than copy-pasting?
  - Although the continuation is straightforward, physical quantities diverge as $\rho \to 0$
  - A prescription is required to "renormalize" the divergences

Glue-on AdS_3 / $T\bar{T}$— updating the dictionary

Kraus, Liu & Marolf, 1801.02714 [11] – "Cutoff AdS_3 versus the $T\overline{T}$ deformation"

- Firstly, matching energy momentum (Brown-York) & the flow equations:

$$T_{ij} = \frac{\sigma_{\zeta_c}}{8\pi G}\left(K_{ij} - K\gamma_{ij} + \frac{1}{\ell|\zeta_c|}\gamma_{ij}\right), \qquad \sigma_{\zeta_c} = \frac{|\zeta_c|}{\zeta_c} = \pm 1, \qquad (12)$$

The field theory metric $\gamma_{ij} = \zeta_c h_{ij}$, where h_{ij} is the induced metric.

  - γ_{ij} is always positive-definite, while h_{ij} becomes negative-definite for the glue-on region. This discrepancy is the origin of the $|\zeta_c|$.

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Glue-on AdS_3 — beyond the infinity

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

- The geometry is foliated by constant ζ surfaces $\mathcal{N}_\zeta$:

$$\begin{aligned} ds^2 &= \ell^2\left(\frac{d\rho^2}{4\rho^2} + \frac{\left(du + \rho\,\bar{\mathcal{L}}(v)\,dv\right)\left(dv + \rho\,\mathcal{L}(u)\,du\right)}{\rho}\right) \\ &= \frac{1}{\zeta}\gamma_{ij}dx^i dx^j + n_\mu n_\nu dx^\mu dx^\nu, \qquad \frac{1}{\zeta} \equiv \ell^{-2} g_{\varphi\varphi} \end{aligned} \tag{14}$$

  The $T\bar{T}$ deformed theory lives on $\mathcal{N}_{\zeta_c}$ with metric γ_{ij}.

- Treatment for the singularity at $\zeta \to 0^\pm$: introduce $\mathcal{N}_{\zeta=\pm\epsilon}$ and glue them together ("glue-on"); exclude the $-\epsilon < \zeta < \epsilon$ region until finally sending $\epsilon \to 0$.

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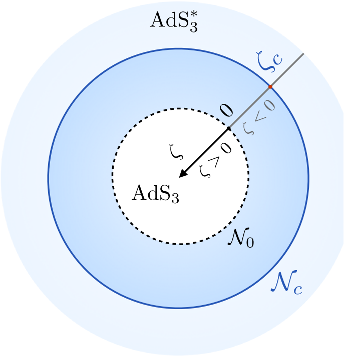

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Check: $T\bar{T}$ deformed charges & partition functions

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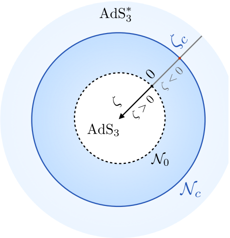

Figure: Glue-on AdS_3

Top-down view of the Poincaré disk

- AdS/CFT: weak / strong duality
  - *quantum* quantities on the CFT side can be computed by (semi-)classical geometry
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- Demanding the extended geometry to be non-singular re-produces bounds on the $T\bar{T}$ deformed theory, e.g.

$$\zeta_c \geq -1 \quad \Leftrightarrow \quad \mu \leq \frac{3\ell^2}{c} \tag{15}$$

$\zeta_c = -1$ is a horizon of the glue-on geometry, where $\det g_{\mu\nu} = 0$.

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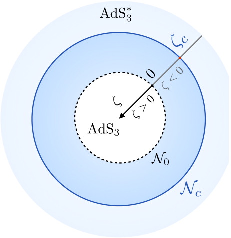

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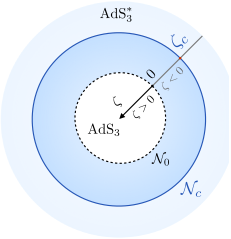

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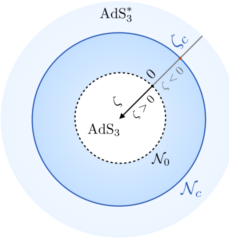

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$$ds_c^2 = \ell^2(-dt'^2 + d\varphi'^2), \qquad \varphi' \sim \varphi' + 2\pi. \tag{16}$$

- This is realized by the *state-dependent* map of coordinates

$$dt' = \sqrt{\left(1 - \zeta_c (T_u + T_v)^2\right)\left(1 - \zeta_c (T_u - T_v)^2\right)}\, dt,$$
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- Many consequences: correct charges, signal propagation speed $v'_\pm \equiv \pm\frac{d\varphi'}{dt'}$, and thermodynamics when Wick rotated:

$$T_L(\mu)\,T_R(\mu) \le -\frac{1}{4\pi^2\ell^2\zeta_c} = \frac{3}{4\pi^2 c\mu} = T_H(\mu)^2, \quad \mu > 0 \qquad (17)$$

$T_{L,R}$ are temperatures associated with $u', v' = \varphi' \pm t'$.

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Check: $T\bar{T}$ partition functions from glue-on AdS

$T\bar{T}$ partition functions from bulk on-shell actions

Caputa, Datta, Jiang & Kraus, 2011.04664 [25]

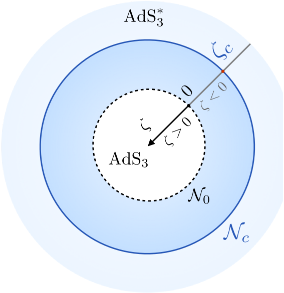

Figure: Glue-on AdS_3

Top-down view of the Poincaré disk

- Bulk: weakly coupled gravity, the partition function is approximated by the on-shell gravity action of the dominant saddle:

$$Z_{T\bar{T}}(\mu) = \mathcal{Z}(\zeta_c) \approx e^{-I(\zeta_c)} \tag{18}$$

- $I(\zeta_c)$ is roughly the volume of the geometry, which diverges at $\zeta \to 0^{\pm}$. We apply a natural extension of *holographic renormalization* to remove the divergence.
- $I = -\log \mathcal{Z}$ satisfies the $T\bar{T}$ flow (3). This is non-trivial because we've packaged the quantum corrections of the boundary theory.

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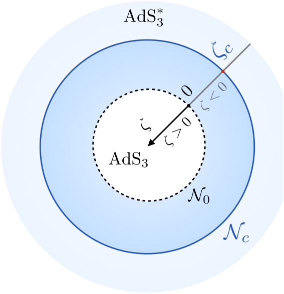

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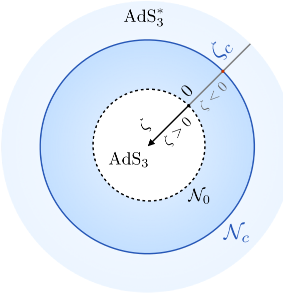

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Comparison with field theory analysis

Datta & Jiang, 1806.07426 [26] and *Apolo, Song & Yu*, 2301.04153 [27]

- **Torus:** Large c, *modular invariance* & sparseness of the "light" spectrum [26, 27], c.f. *Hartman, Keller & Stoica*, 1405.5137 [28]

$$\log Z_{T\bar{T}}(\mu) \approx \begin{cases} -\frac{1}{2}\left(\beta_L + \beta_R\right) R E_{\text{vac}}(\mu), & \beta_L \beta_R > 1, \\ -2\pi^2 \left(\frac{1}{\beta_L} + \frac{1}{\beta_R}\right) R E_{\text{vac}}\left(\frac{4\pi^2}{\beta_L \beta_R}\mu\right), & \beta_L \beta_R < 1, \end{cases}$$

- **Sphere:** maximally symmetric, *Donnelly & Shyam*, 1806.07444 [29]

$$\langle T_{ij}\rangle = -\frac{1}{4\pi\mu}\left(1 - \sqrt{1 - \frac{c\mu}{3L^2}}\right)\gamma_{ij}. \tag{19}$$

Sphere partition functions

Donnelly & Shyam, 1806.07444 [29] and *Li*, 2012.14414 [30]

- Given the explicit stress tensor $\langle T_{ij}\rangle \propto \gamma_{ij}$, the flow equations:

$$\partial_\mu \log Z_{T\bar{T}}(\mu) = 8\pi \int d^2x \sqrt{\gamma}\, \langle T\bar{T}\rangle$$

$$-L\,\partial_L \log Z_{T\bar{T}}(\mu) = \int d^2x \sqrt{\gamma}\, \langle T^i_i\rangle$$

- ... admit the general solution with a μ-*independent* integration a:

$$\log Z_{T\bar{T}}(\mu, a) = \frac{c}{3}\log\left[\frac{L}{a}\left(1+\sqrt{1-\frac{c\mu}{3L^2}}\right)\right] - \frac{L^2}{\mu}\sqrt{1-\frac{c\mu}{3L^2}} + \frac{L^2}{\mu}$$

- *Donnelly & Shyam*, 1806.07444 [29] is recovered with $a = \sqrt{c|\mu|/3}$ but one can also take $a = \epsilon$, thus decoupling the energy (RG) scale ϵ with the deformation μ.

Gravitational actions: with counterterms

Donnelly & Shyam, 1806.07444 [29] and *Li*, 2012.14414 [30]

- Gravitational actions, with counterterms:

$$I_{\mathcal{M}}(\zeta_1,\zeta_2) := -\frac{1}{16\pi G}\int_{\zeta_1}^{\zeta_2} d\zeta \int d^2x\sqrt{g}\,(R+2\ell^{-2}),$$

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  ... consistent with the Brown-York stress tensor T_{ij}, fully renormalized at $\zeta \to 0^{\pm}$.

- The lack of diff invariance of the $\log|\zeta|$ counterterm is known to be a reflection of the Weyl anomaly: *Henningson & Skenderis, de Haro, Solodukhin & Skenderis, Papadimitriou.*

Gravitational actions: with counterterms

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- The $\log$ counterterm makes a crucial contribution to the on-shell action:

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  It guarantees that the partition function is compatible with the $T\bar{T}$ differential equation — not the case for *Donnelly & Shyam*.

- In general the space of deformed theories are parametrized by (μ, a), where a is the length scale (inverse energy scale). In many contexts they are naturally related: $a=\sqrt{c|\mu|/3}$, but a can be tuned by RG.

Summary & future directions

Glue-on AdS_3 / $T\bar{T}$ deformed CFT_2

Apolo, Hao, Lai & Song, 2303.04836 [20] – "Glue-on AdS holography for $T\bar{T}$-deformed CFTs"

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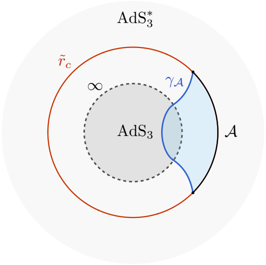

Figure: RT for Glue-on AdS_3
Top-down view of the Poincaré disk

- Holographic proposal for $T\bar{T}$-deformed CFTs with $\mu \in \mathbb{R}$.
- As evidence, we show that the $T\bar{T}$ trace flow equation, the spectrum on the cylinder, and the partition function on the torus and the sphere, among other results, can all be reproduced from bulk calculations in glue-on AdS_3.
- We hope to understand the entanglement structure of $T\bar{T}$ deformed theories from bulk geometry.

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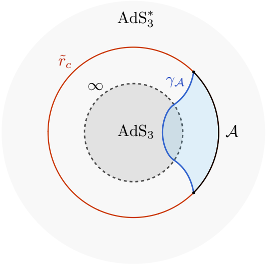

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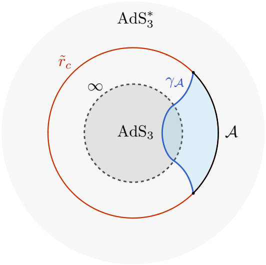

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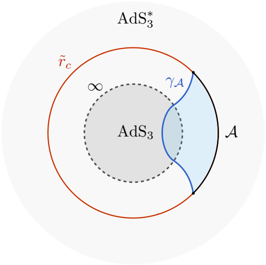

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  - *He & Sun*, 2301.04435 [35], *He, Yang, Zhang & Zhao*, 2305.10984 [36], *Tian*, 2306.01258 [37], and *Hou, He & Jiang*, 2306.07784 [38]

Thank you!

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References I

- Clara Aldegunde. *Knitting space–time out of quantum entanglement*. **September 13, 2022**. URL: https://physicsworld.com/a/knitting-space-time-out-of-quantum-entanglement/.

- Juan Martin Maldacena. *The Large N limit of superconformal field theories and supergravity*. *Adv. Theor. Math. Phys.* **2**:231–252, **1998**. arXiv: hep-th/9711200.

- Charles Fefferman & C. Robin Graham. *The ambient metric*. *Ann. Math. Stud.* **178**:1–128, **2011**. arXiv: 0710.0919 [math.DG].

- Maximo Banados. *Three-dimensional quantum geometry and black holes*. *AIP Conf. Proc.* **484**(1):147–169, **1999**. H. Falomir, R. E. Gamboa Saravi & F. A. Schaposnik, editors. arXiv: hep-th/9901148.

- Ofer Aharony, Steven S. Gubser, Juan Martin Maldacena, Hirosi Ooguri & Yaron Oz. *Large N field theories, string theory and gravity*. *Phys. Rept.* **323**:183–386, **2000**. arXiv: hep-th/9905111.

References II

Maximo Banados, Claudio Teitelboim & Jorge Zanelli. *The Black hole in three-dimensional space-time*. *Phys. Rev. Lett.* **69**:1849–1851, **1992**. arXiv: hep-th/9204099 [hep-th].

Wei Cui, Hongfei Shu, Wei Song & Juntao Wang. *Correlation Functions in the TsT/$T\bar{T}$ Correspondence*. **April 2023**. arXiv: 2304.04684 [hep-th].

Alexander B. Zamolodchikov. *Expectation value of composite field T anti-T in two-dimensional quantum field theory*. **January 2004**. arXiv: hep-th/0401146.

F. A. Smirnov & A. B. Zamolodchikov. *On space of integrable quantum field theories*. *Nucl. Phys.* **B915**:363–383, **2017**. arXiv: 1608.05499 [hep-th].

Andrea Cavaglià, Stefano Negro, István M. Szécsényi & Roberto Tateo. *$T\bar{T}$-deformed 2D Quantum Field Theories*. *JHEP*. **10**:112, **2016**. arXiv: 1608.05534 [hep-th].

Per Kraus, Junyu Liu & Donald Marolf. *Cutoff AdS$_3$ versus the $T\overline{T}$ deformation*. *JHEP*. **07**:027, **2018**. arXiv: 1801.02714 [hep-th].

John Cardy. *$T\bar{T}$ deformation of correlation functions*. *JHEP*. **12**:160, **2019**. arXiv: 1907.03394 [hep-th].

References III

- Sergei Dubovsky, Victor Gorbenko & Mehrdad Mirbabayi. *Asymptotic fragility, near AdS$_2$ holography and $T\overline{T}$*. *JHEP*. **09**:136, **2017**. arXiv: 1706.06604 [hep-th].
- Idse Heemskerk & Joseph Polchinski. *Holographic and Wilsonian Renormalization Groups*. *JHEP*. **06**:031, **2011**. arXiv: 1010.1264 [hep-th].
- Sergei Dubovsky, Raphael Flauger & Victor Gorbenko. *Solving the Simplest Theory of Quantum Gravity*. *JHEP*. **09**:133, **2012**. arXiv: 1205.6805 [hep-th].
- Sergei Dubovsky, Victor Gorbenko & Mehrdad Mirbabayi. *Natural Tuning: Towards A Proof of Concept*. *JHEP*. **09**:045, **2013**. arXiv: 1305.6939 [hep-th].
- Lauren McGough, Márk Mezei & Herman Verlinde. *Moving the CFT into the bulk with $T\overline{T}$*. *JHEP*. **04**:010, **2018**. arXiv: 1611.03470 [hep-th].
- Monica Guica & Ruben Monten. *$T\bar{T}$ and the mirage of a bulk cutoff*. *SciPost Phys.* **10**(2):024, **2021**. arXiv: 1906.11251 [hep-th].
- Per Kraus, Ruben Monten & Richard M. Myers. *3D Gravity in a Box*. **March 2021**. arXiv: 2103.13398 [hep-th].

References IV

Luis Apolo, Peng-Xiang Hao, Wen-Xin Lai & Wei Song. *Glue-on AdS holography for $T\bar{T}$-deformed CFTs*. **March 2023**. arXiv: 2303.04836 [hep-th].

Vivek Iyer & Robert M. Wald. *Some properties of Noether charge and a proposal for dynamical black hole entropy*. *Phys. Rev. D*. **50**:846–864, **1994**. arXiv: gr-qc/9403028.

Glenn Barnich & Friedemann Brandt. *Covariant theory of asymptotic symmetries, conservation laws and central charges*. *Nucl. Phys. B*. **633**:3–82, **2002**. arXiv: hep-th/0111246.

Amit Giveon, Nissan Itzhaki & David Kutasov. $\mathrm{T}\overline{\mathrm{T}}$ *and LST*. *JHEP*. **07**:122, **2017**. arXiv: 1701.05576 [hep-th].

Luis Apolo, Stephane Detournay & Wei Song. *TsT, $T\bar{T}$ and black strings*. *JHEP*. **06**:109, **2020**. arXiv: 1911.12359 [hep-th].

Pawel Caputa, Shouvik Datta, Yunfeng Jiang & Per Kraus. *Geometrizing $T\overline{T}$*. *JHEP*. **03**:140, **2021**. arXiv: 2011.04664 [hep-th]. [Erratum: JHEP 09, 110 (2022)].

References V

- Shouvik Datta & Yunfeng Jiang. *$T\bar{T}$ deformed partition functions.* JHEP. **08**:106, **2018**. arXiv: 1806.07426 [hep-th].
- Luis Apolo, Wei Song & Boyang Yu. *On the universal behavior of $T\bar{T}$-deformed CFTs: single and double-trace partition functions at large c.* **January 2023**. arXiv: 2301.04153 [hep-th].
- Thomas Hartman, Christoph A. Keller & Bogdan Stoica. *Universal Spectrum of 2d Conformal Field Theory in the Large c Limit.* JHEP. **09**:118, **2014**. arXiv: 1405.5137 [hep-th].
- William Donnelly & Vasudev Shyam. *Entanglement entropy and $T\overline{T}$ deformation.* Phys. Rev. Lett. **121**(13):131602, **2018**. arXiv: 1806.07444 [hep-th].
- Yi Li. *Comments on large central charge $T\bar{T}$ deformed conformal field theory and cutoff AdS holography.* **December 2020**. arXiv: 2012.14414 [hep-th].
- M. Henningson & K. Skenderis. *The Holographic Weyl anomaly.* JHEP. **07**:023, **1998**. arXiv: hep-th/9806087.

References VI

Sebastian de Haro, Sergey N. Solodukhin & Kostas Skenderis. *Holographic reconstruction of space-time and renormalization in the AdS / CFT correspondence*. Commun. Math. Phys. **217**:595–622, **2001**. arXiv: hep-th/0002230.

Ioannis Papadimitriou. *Holographic renormalization as a canonical transformation*. JHEP. **11**:014, **2010**. arXiv: 1007.4592 [hep-th].

Aitor Lewkowycz, Junyu Liu, Eva Silverstein & Gonzalo Torroba. *$T\overline{T}$ and EE, with implications for (A)dS subregion encodings*. JHEP. **04**:152, **2020**. arXiv: 1909.13808 [hep-th].

Miao He & Yuan Sun. *Holographic entanglement entropy in $T\overline{T}$-deformed AdS_3*. **January 2023**. arXiv: 2301.04435 [hep-th].

Song He, Jie Yang, Yu-Xuan Zhang & Zi-Xuan Zhao. *Pseudo entropy of primary operators in $T\bar{T}/J\bar{T}$-deformed CFTs*. **May 2023**. arXiv: 2305.10984 [hep-th].

Jia Tian. *On-shell action and Entanglement entropy of $T\bar{T}$-deformed Holographic CFTs*. **June 2023**. arXiv: 2306.01258 [hep-th].

References VII

Jue Hou, Miao He & Yunfeng Jiang. *$T\bar{T}$-deformed Entanglement Entropy for Integrable Quantum Field Theory*. **June 2023**. arXiv: 2306.07784 [hep-th].